Mathematics

Year 12 Summer Tasks

Welcome to Mathematics at St Peter's. The purpose of this booklet is to introduce you to the subject that you have chosen to study with us at A-Level.

The Mathematics department aims to develop your logic and problem solving skills whist introducing you to more complex mathematical material through the study of Pure mathematics and showing the versatility of this subject through the applications of Mechanics and Statistics.

Successful study in the sixth form is all about developing the right skills, preparation, independent learning and self-motivation. We will do all we can to support you in your learning and would like you to do the same. In preparation for the first term of study please complete the following:

Task

Using the "GCSE to A-Level Transition tasks" filled with the most important GCSE topics please spend some time working through the questions so that when you arrive with us in September you can make the best possible start to your studies.
If you are currently at St Peter's these are the tasks that have been set through teams and DrFrostmaths.com.

Layout and structure of your workings is an important skill to develop at A Level, as well as the ability to self-study outside of the subject. Therefore we ask that you make sure you are demonstrating methods where necessary and use the videos provided to work on these key skills.

For current student work will be marked weekly, external students can give their work to their teacher when they arrive.

Equipment

In order that you are prepared for your studies next year please have the following things ready for the start of term.

An A4 folder for your notes, class handouts and written assignments. *It is important that folders are brought to each lesson so teachers can keep a track on your progress. We have found that many students buy a relatively think folder to bring to every lesson, but have a larger lever ached file at home to transfer older notes into to avoid overloading on a daily basis.*

Dividers for folders to keep different topic notes separate, perhaps between your two teachers.

A4 5mm square (ideally) of A4 lined pad of paper

A specific exam eligible calculator will be needed. We suggest the Casio Classwiz fx-991EX. *They are available for around £20 but can be found cheaper.*

Enjoy your summer – we look forward to working with you in Year 12

ST PETER'S CATHOLIC SCHOOL

Year 11 Preparing for A Level Maths:
Helpful Videos and Resources for Working from Home

	Topics to Prepare for A-level Maths	Cortbett Maths Videos and Resources (No subscription required)
1	Manipulating algebraic expressions	15 - Algebra: expanding three brackets 21 - Algebraic Fractions: addition 22 - Algebraic Fractions: division 23 - Algebraic Fractions: Multiplication
2	Surds	305 - Surds: intro, rules, simplifying 307 - Surds: rationalising denominators 308 - Surds: expanding brackets
3	Rules of indices	173 - Indices: fractional 174 - Indices: laws of 175 - Indices: negative
4	Factorising expressions	119a - Factorisation: splitting the middle 120 - Factorisation: difference of 2 squares 24 - Algebraic Fractions: Simplifying
5	Completing the square	10 - Algebra: completing the square
6	Solving quadratic equations	266 - Quadratics: solving (factorising) 267 - Quadratics: formula 267a - Quadratics: solving (completing the square)
7	Sketching quadratic graphs	265 - Quadratic graphs: sketching using key points 371 - Quadratic graph (completing the square)
8	Solving linear simultaneous equations	295 - Simultaneous equations (elimination) 296 - Simultaneous equations (substitution, both linear)
9	Solving quadratic simultaneous equations	12 - Algebra: equation of a circle 298 - Simultaneous equations (advanced)
10	Solving simultaneous equations graphically	297 - Simultaneous equations (graphical)
11	Linear inequalities	180 - Inequalities: graphical y>a or x>a 181 - Inequalities: graphical y>x+a 182 - Inequalities: region
12	Quadratic inequalities	378 - Inequalities: quadratic
13	Sketching cubic and reciprocal graphs	344 - Types of graph: cubics 346 - Types of graph: reciprocal
14	Translating graphs	323 - Transformations of graphs
15	Straight line graphs	194 - Linear graphs: find equation of a line 195 - Linear graphs: equation through 2 points
16	Parallel and perpendicular lines	196 - Linear graphs: parallel lines 197 - Linear graphs: perpendicular lines 372 - Equation of a Tangent to a Circle
17	Pythagoras' Theorem	259 - Pythagoras: 3D 260 - Pythagoras: rectangles/isosceles triangles 263 - Pythagoras: distance points
18	Direct and inverse proportion	254 - Proportion: direct 255 - Proportion: inverse
19	Circle theorems	64 - Circle theorems – theorems 65 - Circle theorems – examples

20	Trigonometry	332 - Trigonometry: 3D 338 - Trigonometry: Sine graph 339 - Trigonometry: Cosine graph 340 - Trigonometry: Tangent graph 333 - Trigonometry: sine rule (sides) 334 - Trigonometry: sine rule (angles) 334a - Trigonometry: sine rule (ambiguous case) 335 - Trigonometry: cosine rule (sides) 336 - Trigonometry: cosine rule (angles)
21	Rearranging equations	111 - Equations: involving fractions 112 - Equations: fractional advanced 112 - Equations: cross multiplication
22	Volume and surface area of 3D shapes	359 - Volume: cone 360 - Volume: pyramid 360a - Volume: Frustum 361 - Volume: sphere 313 - Surface area: sphere 314 - Surface area: cone 315 - Surface area: cylinders
23	Area under a graph and gradients	389 - Area under a Graph 390a - Instantaneous Rate of Change

Expanding brackets and simplifying expressions

Corbettmaths.com videos of use:

15 - Algebra: expanding three brackets

21 - Algebraic Fractions: addition

22 - Algebraic Fractions: division

23 - Algebraic Fractions: Multiplication

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

Watch out!

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

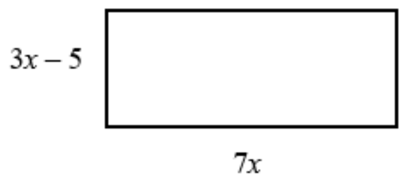
6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x, for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

e $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$

i $(3x + 4y)(5y + 6x)$ **j** $(x + 5)^2$

k $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$ **b** $\left(x + \frac{1}{x}\right)^2$

Answers

1 **a** $6x - 3$ **b** $-10pq - 8q^2$
c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \dfrac{2}{x^2}$ **b** $x^2 + 2 + \dfrac{1}{x^2}$

Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Corbettmaths.com videos of use:
305 - Surds: intro, rules, simplifying
307 - Surds: rationalising denominators
308 - Surds: expanding brackets

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$ $= 5\sqrt{2}$	**3** Use $\sqrt{25} = 5$

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $=\sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2=49$
$=7-2$ $=5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $=-\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2}=0$

Example 4 Rationalise $\dfrac{1}{\sqrt{3}}$

$\dfrac{1}{\sqrt{3}}=\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$=\dfrac{1\times\sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9}=3$
$=\dfrac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\dfrac{\sqrt{2}}{\sqrt{12}}$

$\dfrac{\sqrt{2}}{\sqrt{12}}=\dfrac{\sqrt{2}}{\sqrt{12}}\times\dfrac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$=\dfrac{\sqrt{2}\times\sqrt{4\times3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
$=\dfrac{2\sqrt{2}\sqrt{3}}{12}$	**3** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **4** Use $\sqrt{4}=2$
$=\dfrac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\dfrac{2}{12}$ simplifies to $\dfrac{1}{6}$

Example 6 Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \dfrac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$	**2** Expand the brackets
$= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \dfrac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
$= 3\sqrt{5}-6$	

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$
c $4\sqrt{3}$ **d** $5\sqrt{7}$
e $10\sqrt{3}$ **f** $2\sqrt{7}$
g $6\sqrt{2}$ **h** $9\sqrt{2}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$
c $3\sqrt{2}$ **d** $\sqrt{3}$
e $6\sqrt{7}$ **f** $5\sqrt{3}$

3 **a** -1 **b** $9-\sqrt{3}$
c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$
c $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$
e $\sqrt{2}$ **f** $\sqrt{5}$
g $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Corbettmaths.com videos of use:

173 - Indices: fractional

174 - Indices: laws of

175 - Indices: negative

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \dfrac{1}{4^2}$ $= \dfrac{1}{16}$	**1** Use the rule $a^{-m} = \dfrac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\dfrac{6x^5}{2x^2}$

$\dfrac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\dfrac{x^3 \times x^5}{x^4}$

$\dfrac{x^3 \times x^5}{x^4} = \dfrac{x^{3+5}}{x^4} = \dfrac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\dfrac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\dfrac{1}{3x}$ as a single power of x

$\dfrac{1}{3x} = \dfrac{1}{3}x^{-1}$	Use the rule $\dfrac{1}{a^m} = a^{-m}$, note that the fraction $\dfrac{1}{3}$ remains unchanged

Example 8 Write $\dfrac{4}{\sqrt{x}}$ as a single power of x

$\dfrac{4}{\sqrt{x}} = \dfrac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\dfrac{1}{a^m} = a^{-m}$

edexcel

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

c $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3y^2}{14x^5y}$

e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\dfrac{\left(2x^2\right)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

> **Watch out!**
>
> Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x}}$ **f** $\dfrac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of x.

a $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

Answers

1	**a**	1	**b**	1	**c**	1	**d**	1
2	**a**	7	**b**	4	**c**	5	**d**	2
3	**a**	125	**b**	32	**c**	343	**d**	8
4	**a**	$\frac{1}{25}$	**b**	$\frac{1}{64}$	**c**	$\frac{1}{32}$	**d**	$\frac{1}{36}$
5	**a**	$\frac{3x^3}{2}$	**b**	$5x^2$				
	c	$3x$	**d**	$\frac{y}{2x^2}$				
	e	$y^{\frac{1}{2}}$	**f**	c^{-3}				
	g	$2x^6$	**h**	x				
6	**a**	$\frac{1}{2}$	**b**	$\frac{1}{9}$	**c**	$\frac{8}{3}$		
	d	$\frac{1}{4}$	**e**	$\frac{4}{3}$	**f**	$\frac{16}{9}$		
7	**a**	x^{-1}	**b**	x^{-7}	**c**	$x^{\frac{1}{4}}$		
	d	$x^{\frac{2}{5}}$	**e**	$x^{-\frac{1}{3}}$	**f**	$x^{-\frac{2}{3}}$		
8	**a**	$\frac{1}{x^3}$	**b**	1	**c**	$\sqrt[5]{x}$		
	d	$\sqrt[5]{x^2}$	**e**	$\frac{1}{\sqrt{x}}$	**f**	$\frac{1}{\sqrt[4]{x^3}}$		
9	**a**	$5x^{\frac{1}{2}}$	**b**	$2x^{-3}$	**c**	$\frac{1}{3}x^{-4}$		
	d	$2x^{-\frac{1}{2}}$	**e**	$4x^{-\frac{1}{3}}$	**f**	$3x^0$		
10	**a**	$x^3 + x^{-2}$	**b**	$x^3 + x$	**c**	$x^{-2} + x^{-7}$		

Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Corbettmaths.com videos of use:

119a - Factorisation: splitting the middle

120 - Factorisation: difference of 2 squares

24 - Algebraic Fractions: Simplifying

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$

b $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

Hint

Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$

b $x^2 + 5x - 14$

c $x^2 - 11x + 30$

d $x^2 - 5x - 24$

e $x^2 - 7x - 18$

f $x^2 + x - 20$

g $x^2 - 3x - 40$

h $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$

b $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$

b $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$

d $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$

f $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$

b $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$

d $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$

f $\dfrac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$

b $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

c $\dfrac{4-25x^2}{10x^2-11x-6}$

d $\dfrac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2+10x+25}$

8 Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$

c $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$

c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$

e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$

g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$

c $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$

c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$

e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\dfrac{2(x+2)}{x-1}$ **b** $\dfrac{x}{x-1}$

c $\dfrac{x+2}{x}$ **d** $\dfrac{x}{x+5}$

e $\dfrac{x+3}{x}$ **f** $\dfrac{x}{x-5}$

6 **a** $\dfrac{3x+4}{x+7}$ **b** $\dfrac{2x+3}{3x-2}$

c $\dfrac{2-5x}{2x-3}$ **d** $\dfrac{3x+1}{x+4}$

7 $(x + 5)$

8 $\dfrac{4(x+2)}{x-2}$

edexcel

Completing the square

Corbettmaths.com videos of use:

10 - Algebra: completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x-\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x-\frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x-\frac{5}{4}\right)^2 - \frac{17}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ **3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 **4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

c $x^2 - 8x$ **d** $x^2 + 6x$

e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 **a** $(x+2)^2-1$ **b** $(x-5)^2-28$

c $(x-4)^2-16$ **d** $(x+3)^2-9$

e $(x-1)^2+6$ **f** $\left(x+\frac{3}{2}\right)^2-\frac{17}{4}$

2 **a** $2(x-2)^2-24$ **b** $4(x-1)^2-20$

c $3(x+2)^2-21$ **d** $2\left(x+\frac{3}{2}\right)^2-\frac{25}{2}$

3 **a** $2\left(x+\frac{3}{4}\right)^2+\frac{39}{8}$ **b** $3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}$

c $5\left(x+\frac{3}{10}\right)^2-\frac{9}{20}$ **d** $3\left(x+\frac{5}{6}\right)^2+\frac{11}{12}$

4 $(5x+3)^2+3$

Solving quadratic equations by factorisation

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Corbettmaths.com videos of use:
266 - Quadratics: solving (factorising)

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$
c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$
e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$
g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$
i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$
k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$
c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$
e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$
g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

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Solving quadratic equations by completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Corbettmaths.com videos of use:

267a - Quadratics: solving (completing the square)

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 9 + 4 = 0$	
$(x + 3)^2 - 5 = 0$	**2** Simplify.
$(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.

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$2\left(x-\frac{7}{4}\right)^2=\frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x-\frac{7}{4}\right)^2=\frac{17}{16}$	**6** Divide both sides by 2.
$x-\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x=\pm\frac{\sqrt{17}}{4}+\frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x=\frac{7}{4}-\frac{\sqrt{17}}{4}$ or $x=\frac{7}{4}+\frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Corbettmaths.com videos of use:

267 - Quadratics: formula

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3$, $b = -7$, $c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

7 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

8 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

Sketching quadratic graphs

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

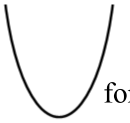
Corbettmaths.com videos of use:

265 - Quadratic graphs: sketching using key points

371 - Quadratic graph (completing the square)

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

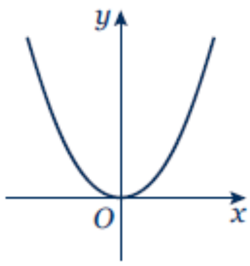

for $a > 0$

for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

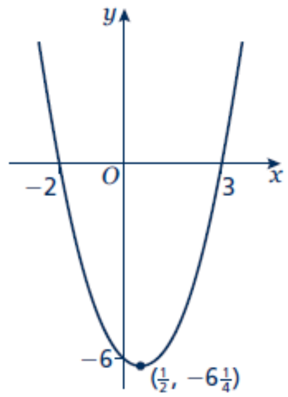
Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	**1** Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0$, $x^2 - x - 6 = 0$	**2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**5** $a = 1$ which is greater than zero, so the graph has the shape: 

(continued on next page)

$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$	**6** To find the turning point, complete the square.
When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ 	**7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

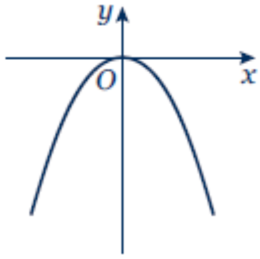
5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

1



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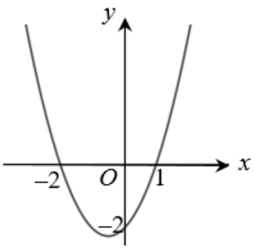

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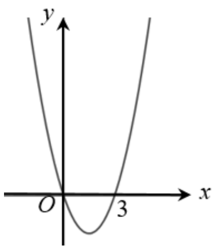

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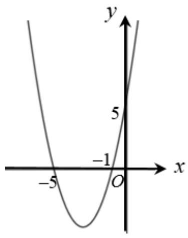

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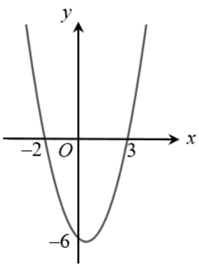

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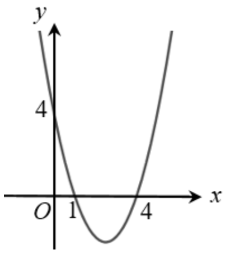

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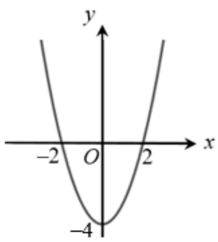

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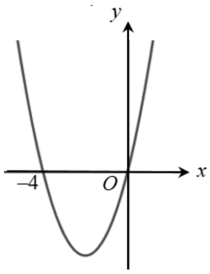

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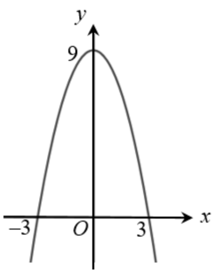

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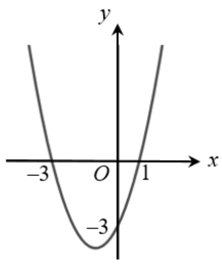

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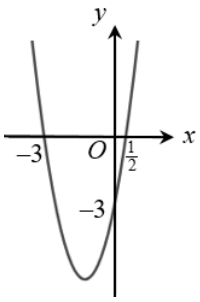
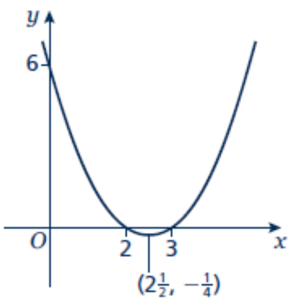


5 **a**



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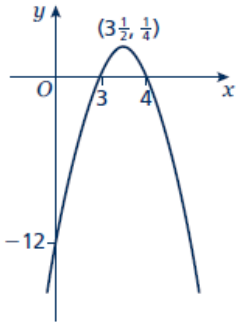

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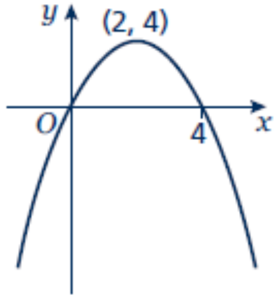

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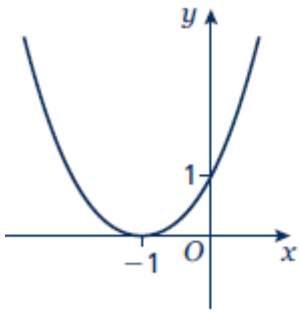


Line of symmetry at $x = -1$.

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Corbettmaths.com videos of use:
295 - Simultaneous equations (elimination)

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$$\begin{array}{rl} & 3x + y = 5 \\ - & \;\;x + y = 1 \\ \hline & 2x \quad\;\; = 4 \end{array}$$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$$\begin{array}{rl} & x + 2y = 13 \\ + & 5x - 2y = \;\;5 \\ \hline & 6x \quad\;\;\; = 18 \end{array}$$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Corbettmaths.com videos of use:

296 - Simultaneous equations (substitution, both linear)

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$ $10x - 48 = -3$	**3** Expand the brackets and simplify.
$10x = 45$ So $x = 4\frac{1}{2}$	**4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneously equations.

7 $y = x - 4$
$2x + 5y = 43$

8 $y = 2x - 3$
$5x - 3y = 11$

9 $2y = 4x + 5$
$9x + 5y = 22$

10 $2x = y - 2$
$8x - 5y = -11$

11 $3x + 4y = 8$
$2x - y = -13$

12 $3y = 4x - 7$
$2y = 3x - 4$

13 $3x = y - 1$
$2y - 2x = 3$

14 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Corbettmaths.com videos of use:

12 - Algebra: equation of a circle

298 - Simultaneous equations (advanced)

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2,\ y = 3$ and $x = -3, y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$ So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**6** To find the value of x, substitute both values of y into one of the original equations.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Answers

1 $x = 1, y = 3$

$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Parallel and perpendicular lines

Corbettmaths.com videos of use:

196 - Linear graphs: parallel lines

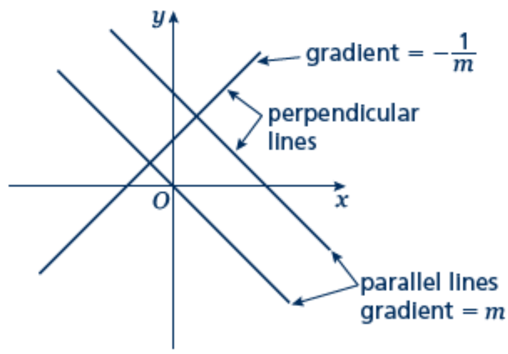
197 - Linear graphs: perpendicular lines

372 - Equation of a Tangent to a Circle

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\dfrac{1}{m} = -\dfrac{1}{2}$ $y = -\dfrac{1}{2}x + c$ $5 = -\dfrac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\dfrac{1}{2}x + 4$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\dfrac{1}{m}$. **2** Substitute $m = -\dfrac{1}{2}$ into $y = mx + c$. **3** Substitute the coordinates (–2, 5) into the equation $y = -\dfrac{1}{2}x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 4$ into $y = -\dfrac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0,\ x_2 = 9,\ y_1 = 5$ and $y_2 = -1$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-5}{9-0}$ $= \dfrac{-6}{9} = -\dfrac{2}{3}$ $-\dfrac{1}{m} = \dfrac{3}{2}$ $y = \dfrac{3}{2}x + c$ Midpoint $= \left(\dfrac{0+9}{2}, \dfrac{5+(-1)}{2}\right) = \left(\dfrac{9}{2}, 2\right)$ $2 = \dfrac{3}{2} \times \dfrac{9}{2} + c$ $c = -\dfrac{19}{4}$ $y = \dfrac{3}{2}x - \dfrac{19}{4}$	**1** Substitute the coordinates into the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** As the lines are perpendicular, the gradient of the perpendicular line is $-\dfrac{1}{m}$. **3** Substitute the gradient into the equation $y = mx + c$. **4** Work out the coordinates of the midpoint of the line. **5** Substitute the coordinates of the midpoint into the equation. **6** Simplify and solve the equation. **7** Substitute $c = -\dfrac{19}{4}$ into the equation $y = \dfrac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, –3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

> **Hint**
> If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L}_1$ passes through the points A and B with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L}_1$ in the form $ax + by + c = 0$

The line $\mathbf{L}_2$ is parallel to the line $\mathbf{L}_1$ and passes through the point C with coordinates (–8, 3).

b Find the equation of $\mathbf{L}_2$ in the form $ax + by + c = 0$

The line $\mathbf{L}_3$ is perpendicular to the line $\mathbf{L}_1$ and passes through the origin.

c Find an equation of $\mathbf{L}_3$

edexcel

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular

d Perpendicular **e** Neither **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

Pearson

Pythagoras' theorem

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Corbettmaths.com videos of use:

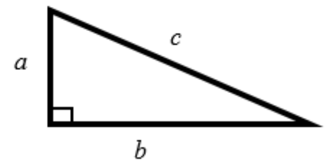
259 - Pythagoras: 3D

260 - Pythagoras: rectangles/isosceles triangles

263 - Pythagoras: distance points

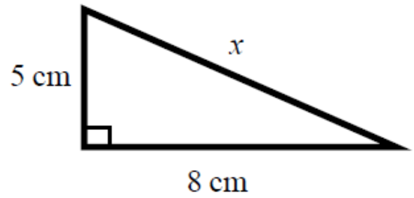
Key points

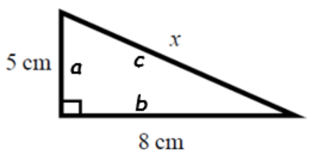
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



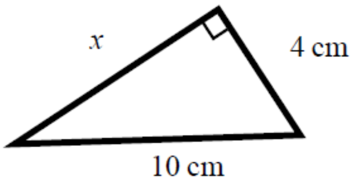
Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.



$c^2 = a^2 + b^2$ 	**1** Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse c and the other two sides a and b.
$x^2 = 5^2 + 8^2$ $x^2 = 25 + 64$ $x^2 = 89$ $x = \sqrt{89}$ $x = 9.433\ 981\ 13...$ $x = 9.43$ cm	**2** Substitute the values of a, b and c into the formula for Pythagoras' theorem. **3** Use a calculator to find the square root. **4** Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length x.
Give your answer in surd form.

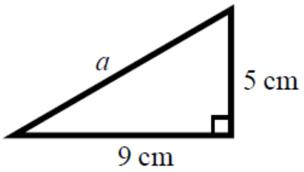


$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21}$ cm	**1** Always start by stating the formula for Pythagoras' theorem. **2** Substitute the values of a, b and c into the formula for Pythagoras' theorem. **3** Simplify the surd where possible and write the units in your answer.

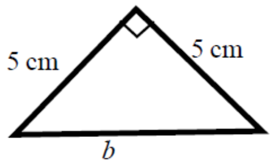
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

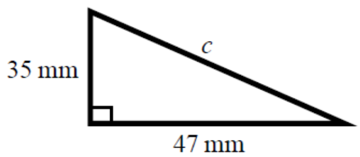
a



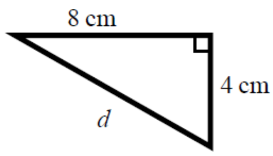
b



c

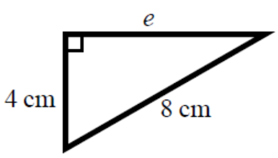


d



2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

a



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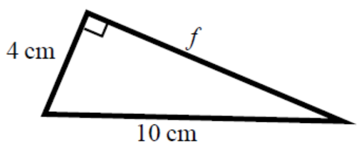

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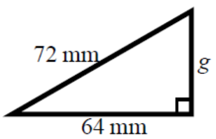
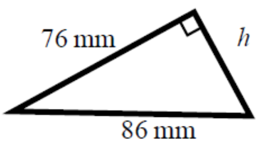
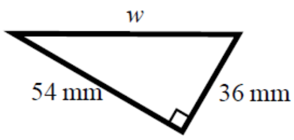


d



3 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

a



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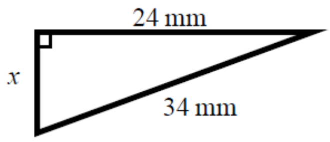
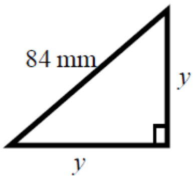
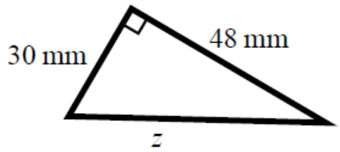


c



d



4 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

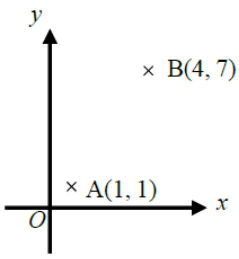
> **Hint**
>
> Draw a sketch of the rectangle.

Extend

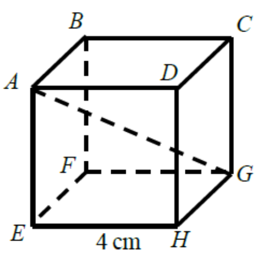
5 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given in the question.

6 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



7 A cube has length 4 cm.
Work out the length of the diagonal *AG*.
Give your answer in surd form.



Answers

1 **a** 10.3 cm **b** 7.07 cm

c 58.6 mm **d** 8.94 cm

2 **a** $4\sqrt{3}$ cm **b** $2\sqrt{21}$ cm

c $8\sqrt{17}$ mm **d** $18\sqrt{5}$ mm

3 **a** $18\sqrt{13}$ mm **b** $2\sqrt{145}$ mm

c $42\sqrt{2}$ mm **d** $6\sqrt{89}$ mm

4 95.3 mm

5 64.0 km

6 $3\sqrt{5}$ units

7 $4\sqrt{3}$ cm

Proportion

Corbettmaths.com videos of use:

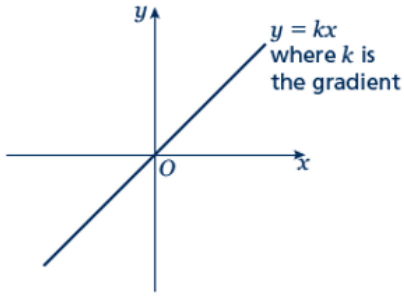
254 - Proportion: direct

255 - Proportion: inverse

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

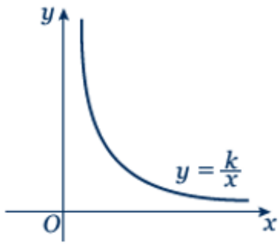
Key points

- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- 'y is directly proportional to x' is written as $y \propto x$.
  If $y \propto x$ then $y = kx$, where k is a constant.
- When x is directly proportional to y, the graph is a straight line passing through the origin.



- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- 'y is inversely proportional to x' is written as $y \propto \frac{1}{x}$.

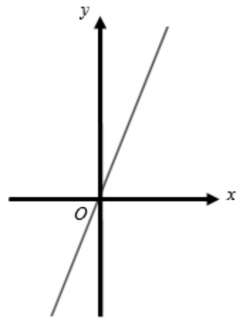
  If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$, where k is a constant.
- When x is inversely proportional to y the graph is the same shape as the graph of $y = \frac{1}{x}$



Examples

Example 1 y is directly proportional to x.
When $y = 16$, $x = 5$.
a Find x when $y = 30$.
b Sketch the graph of the formula.

a $y \propto x$	**1** Write y is directly proportional to x, using the symbol $\propto$.
$y = kx$ $16 = k \times 5$	**2** Write the equation using k. **3** Substitute $y = 16$ and $x = 5$ into $y = kx$.
$k = 3.2$	**4** Solve the equation to find k.
$y = 3.2x$	**5** Substitute the value of k back into the equation $y = kx$.
When $y = 30$, $30 = 3.2 \times x$ $x = 9.375$	**6** Substitute $y = 30$ into $y = 3.2x$ and solve to find x when $y = 30$.

b 	**7** The graph of $y = 3.2x$ is a straight line passing through (0, 0) with a gradient of 3.2.

Example 2 y is directly proportional to x^2.
When $x = 3$, $y = 45$.
a Find y when $x = 5$.
b Find x when $y = 20$.

a $y \propto x^2$	**1** Write y is directly proportional to x^2, using the symbol $\propto$.
$y = kx^2$	**2** Write the equation using k.
$45 = k \times 3^2$	**3** Substitute $y = 45$ and $x = 3$ into $y = kx^2$.
$k = 5$	**4** Solve the equation to find k.
$y = 5x^2$	**5** Substitute the value of k back into the equation $y = kx^2$.
When $x = 5$, $y = 5 \times 5^2$ $y = 125$	**6** Substitute $x = 5$ into $y = 5x^2$ and solve to find y when $x = 5$.
b $20 = 5 \times x^2$ $x^2 = 4$ $x = \pm 2$	**7** Substitute $y = 20$ into $y = 5x^2$ and solve to find x when $y = 4$.

Example 3 P is inversely proportional to Q.
When $P = 100$, $Q = 10$.
Find Q when $P = 20$.

$P \propto \frac{1}{Q}$	**1** Write P is inversely proportional to Q, using the symbol $\propto$.
$P = \frac{k}{Q}$	**2** Write the equation using k.
$100 = \frac{k}{10}$	**3** Substitute $P = 100$ and $Q = 10$.
$k = 1000$	**4** Solve the equation to find k.
$P = \frac{1000}{Q}$	**5** Substitute the value of k into $P = \frac{k}{Q}$
$20 = \frac{1000}{Q}$ $Q = \frac{1000}{20} = 50$	**6** Substitute $P = 20$ into $P = \frac{1000}{Q}$ and solve to find Q when $P = 20$.

Practice

> **Hint**
>
> Substitute the values given for P and h into the formula to calculate k.

1 Paul gets paid an hourly rate. The amount of pay (£P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?

2 x is directly proportional to y.
$x = 35$ when $y = 5$.

- **a** Find a formula for x in terms of y.
- **b** Sketch the graph of the formula.
- **c** Find x when $y = 13$.
- **d** Find y when $x = 63$.

3 Q is directly proportional to the square of Z.
$Q = 48$ when $Z = 4$.

- **a** Find a formula for Q in terms of Z.
- **b** Sketch the graph of the formula.
- **c** Find Q when $Z = 5$.
- **d** Find Z when $Q = 300$.

4 y is directly proportional to the square of x.
$x = 2$ when $y = 10$.

- **a** Find a formula for y in terms of x.
- **b** Sketch the graph of the formula.
- **c** Find x when $y = 90$.

5 B is directly proportional to the square root of C.
$C = 25$ when $B = 10$.

- **a** Find B when $C = 64$.
- **b** Find C when $B = 20$.

6 C is directly proportional to D.
$C = 100$ when $D = 150$.
Find C when $D = 450$.

7 y is directly proportional to x.
$x = 27$ when $y = 9$.
Find x when $y = 3.7$.

8 m is proportional to the cube of n.
$m = 54$ when $n = 3$.
Find n when $m = 250$.

Extend

9 s is inversely proportional to t.

a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t.

b Sketch the graph of the formula.

c Find t when $s = 1$.

10 a is inversely proportional to b.
$a = 5$ when $b = 20$.

a Find a when $b = 50$.

b Find b when $a = 10$.

11 v is inversely proportional to w.
$w = 4$ when $v = 20$.

a Find a formula for v in terms of w.

b Sketch the graph of the formula.

c Find w when $v = 2$.

12 L is inversely proportional to W.
$L = 12$ when $W = 3$.
Find W when $L = 6$.

13 s is inversely proportional to t.
$s = 6$ when $t = 12$.

a Find s when $t = 3$.

b Find t when $s = 18$.

14 y is inversely proportional to x^2.
$y = 4$ when $x = 2$.
Find y when $x = 4$.

15 y is inversely proportional to the square root of x.
$x = 25$ when $y = 1$.
Find x when $y = 5$.

16 a is inversely proportional to b.
$a = 0.05$ when $b = 4$.

a Find a when $b = 2$.

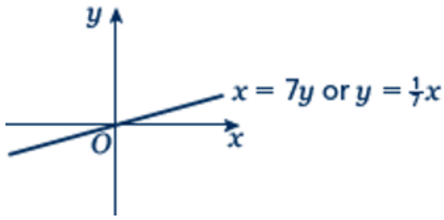
b Find b when $a = 2$.

Answers

1 £77

2 **a** $x = 7y$

b

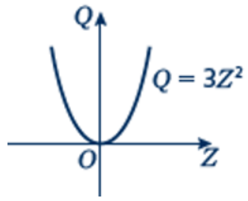


c 91

d 9

3 **a** $Q = 3Z^2$

b

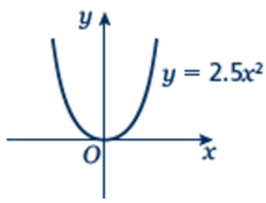


c 75

d ± 10

4 **a** $y = 2.5x^2$

b



c ± 6

5 **a** 16

b 100

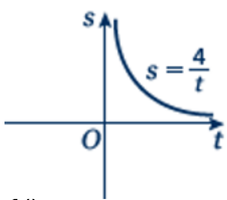
6 300

7 11.1

8 5

9 **a** $s = \dfrac{4}{t}$

b



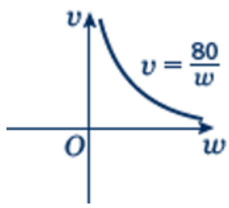
c 4

10 **a** 2

b 10

11 **a** $v = \dfrac{80}{w}$

b



c 40

12 6

13 **a** 24 **b** 4

14 1

15 1

16 **a** 0.1 **b** 0.1

Circle theorems

Corbettmaths.com videos of use:

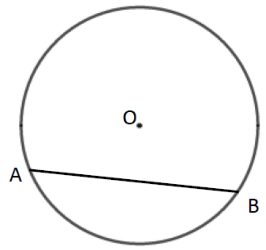
64 - Circle theorems – theorems

65 - Circle theorems – examples

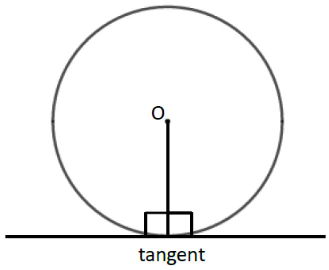
A LEVEL LINKS
Scheme of work: 2b. Circles – equation of a circle, geometric problems on a grid

Key points

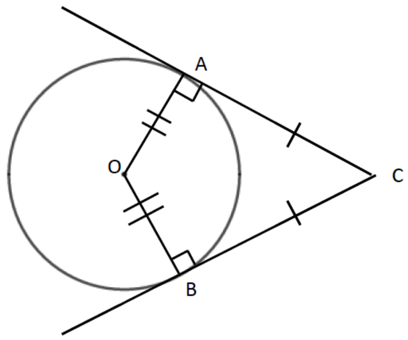
- A chord is a straight line joining two points on the circumference of a circle.
  So AB is a chord.



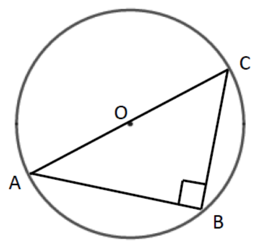
- A tangent is a straight line that touches the circumference of a circle at only one point.
  The angle between a tangent and the radius is 90°.



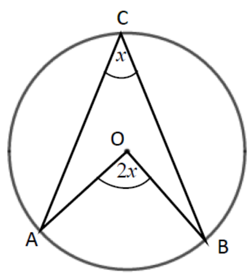
- Two tangents on a circle that meet at a point outside the circle are equal in length.
  So AC = BC.



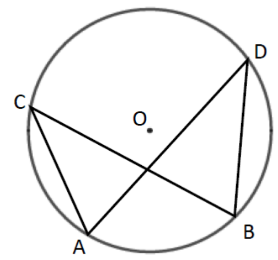
- The angle in a semicircle is a right angle.
  So angle ABC = 90°.



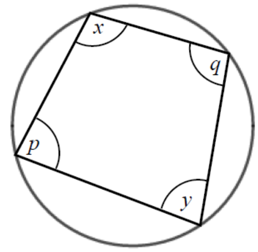
- When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.
  So angle AOB = 2 × angle ACB.



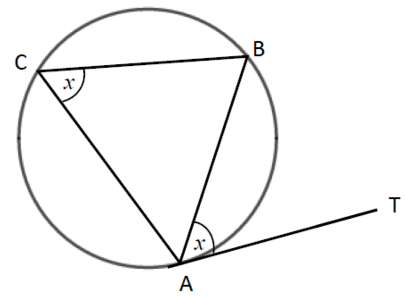
- Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.
  So angle ACB = angle ADB and
  angle CAD = angle CBD.



- A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle.
  Opposite angles in a cyclic quadrilateral total 180°.
  So $x + y = 180°$ and $p + q = 180°$.

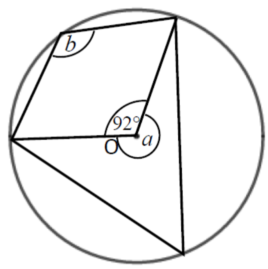


- The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem.
  So angle BAT = angle ACB.



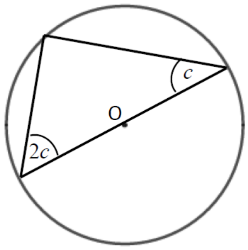
Examples

Example 1 Work out the size of each angle marked with a letter.
Give reasons for your answers.



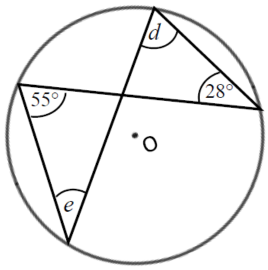
Angle $a = 360° - 92°$ $= 268°$ as the angles in a full turn total 360°.	**1** The angles in a full turn total 360°.
Angle $b = 268° \div 2$ $= 134°$ as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.	**2** Angles a and b are subtended by the same arc, so angle b is half of angle a.

Example 2 Work out the size of the angles in the triangle.
Give reasons for your answers.



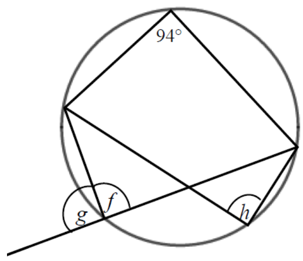
Angles are 90°, $2c$ and c. $90° + 2c + c = 180°$ $90° + 3c = 180°$ $3c = 90°$ $c = 30°$ $2c = 60°$ The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°.	**1** The angle in a semicircle is a right angle. **2** Angles in a triangle total 180°. **3** Simplify and solve the equation.

Example 3 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle d = 55° as angles subtended by the same arc are equal. Angle e = 28° as angles subtended by the same arc are equal.	**1** Angles subtended by the same arc are equal so angle 55° and angle d are equal. **2** Angles subtended by the same arc are equal so angle 28° and angle e are equal.

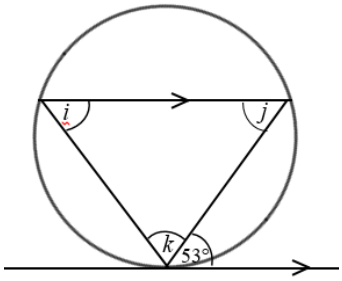
Example 4 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle f = 180° − 94° = 86° as opposite angles in a cyclic quadrilateral total 180°.	**1** Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle f total 180°. *(continued on next page)*

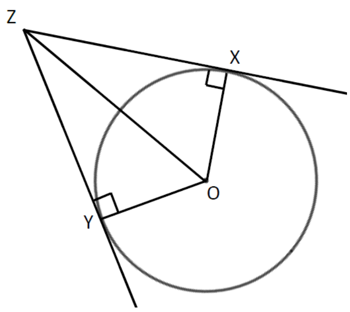
Angle $g = 180° - 86°$ $= 84°$ as angles on a straight line total 180°. Angle h = angle f = 86° as angles subtended by the same arc are equal.	**2** Angles on a straight line total 180° so angle f and angle g total 180°. **3** Angles subtended by the same arc are equal so angle f and angle h are equal.

Example 5 Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle i = 53° because of the alternate segment theorem. Angle j = 53° because it is the alternate angle to 53°. Angle $k = 180° - 53° - 53°$ $= 74°$ as angles in a triangle total 180°.	**1** The angle between a tangent and chord is equal to the angle in the alternate segment. **2** As there are two parallel lines, angle 53° is equal to angle j because they are alternate angles. **3** The angles in a triangle total 180°, so $i + j + k = 180°$.

Example 6 XZ and YZ are two tangents to a circle with centre O. Prove that triangles XZO and YZO are congruent.

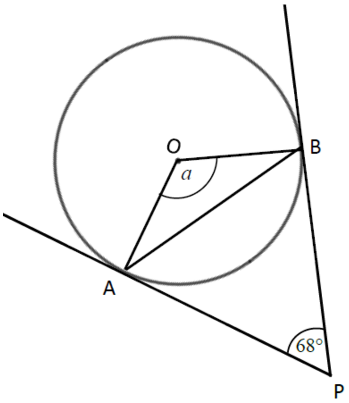


Angle OXZ = 90° and angle OYZ = 90° as the angles in a semicircle are right angles. OZ is a common line and is the hypotenuse in both triangles. OX = OY as they are radii of the same circle. So triangles XZO and YZO are congruent, RHS.	For two triangles to be congruent you need to show one of the following. • All three corresponding sides are equal (SSS). • Two corresponding sides and the included angle are equal (SAS). • One side and two corresponding angles are equal (ASA). • A right angle, hypotenuse and a shorter side are equal (RHS).

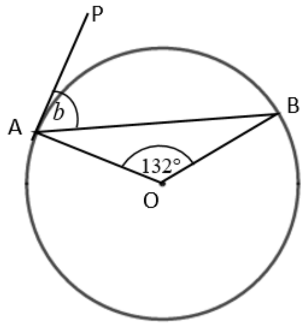
Practice

1 Work out the size of each angle marked with a letter.
Give reasons for your answers.

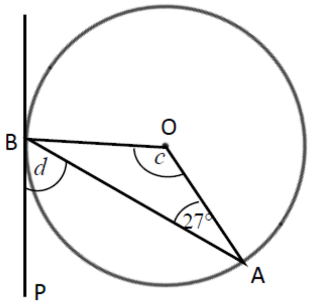
a



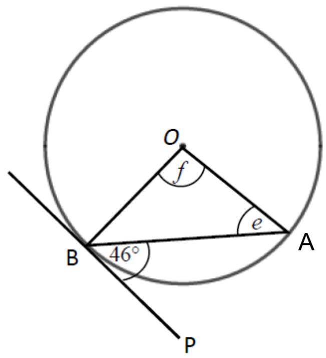
b



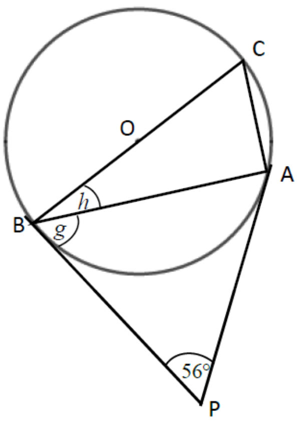
c



d

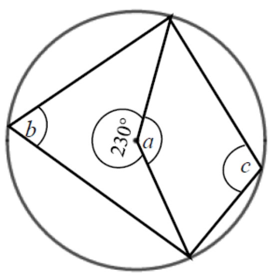


e

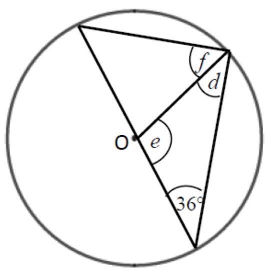


2 Work out the size of each angle marked with a letter.
Give reasons for your answers.

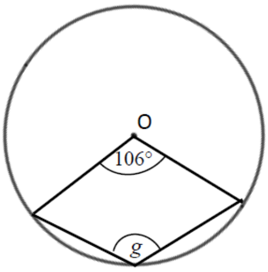
a



b

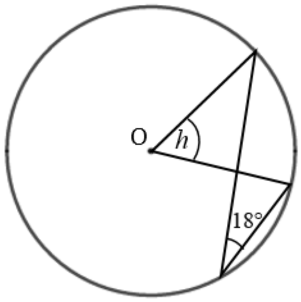


c



> **Hint**
>
> The reflex angle at point O and angle *g* are subtended by the same arc. So the reflex angle is twice the size of angle *g*.

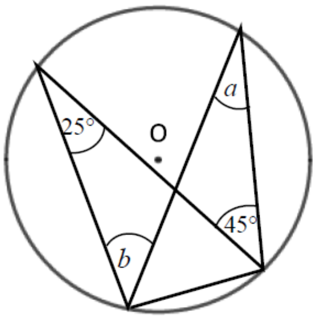
d



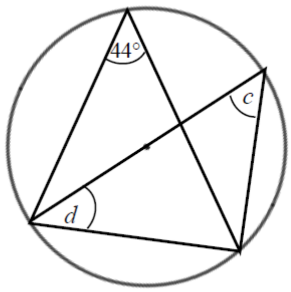
> **Hint**
>
> Angle 18° and angle *h* are subtended by the same arc.

3 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a

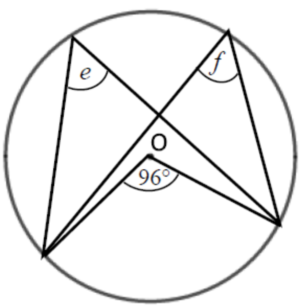


b

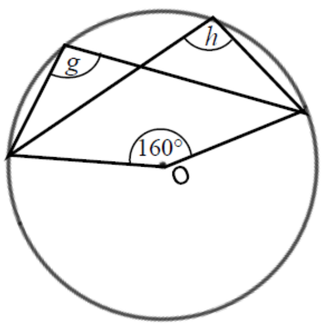


> **Hint**
>
> One of the angles is in a semicircle.

c

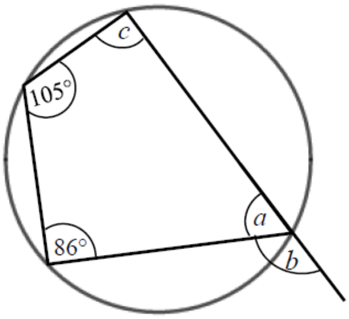


d



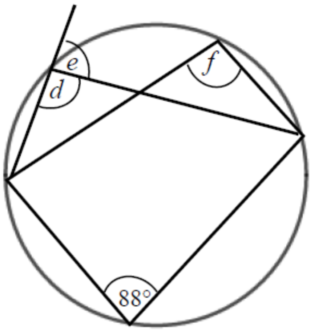
4 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a

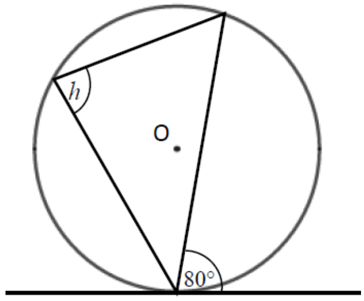


> **Hint**
>
> An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

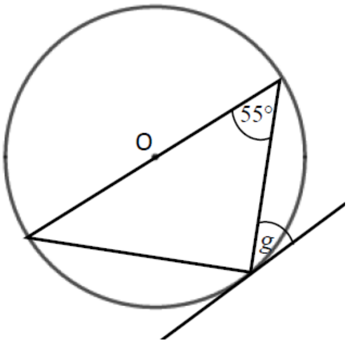
b



c



d



> **Hint**
>
> One of the angles is in a semicircle.

Extend

5 Prove the alternate segment theorem.

Answers

1 **a** $a = 112°$, angle OAP = angle OBP = 90° and angles in a quadrilateral total 360°.

b $b = 66°$, triangle OAB is isosceles, Angle OAP = 90° as AP is tangent to the circle.

c $c = 126°$, triangle OAB is isosceles.
$d = 63°$, Angle OBP = 90° as BP is tangent to the circle.

d $e = 44°$, the triangle is isosceles, so angles *e* and angle OBA are equal. The angle OBP = 90° as BP is tangent to the circle.
$f = 92°$, the triangle is isosceles.

e $g = 62°$, triangle ABP is isosceles as AP and BP are both tangents to the circle.
$h = 28°$, the angle OBP = 90°.

2 **a** $a = 130°$, angles in a full turn total 360°.
$b = 65°$, the angle at the centre of a circle is twice the angle at the circumference.
$c = 115°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 36°$, isosceles triangle.
$e = 108°$, angles in a triangle total 180°.
$f = 54°$, angle in a semicircle is 90°.

c $g = 127°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.

d $h = 36°$, the angle at the centre of a circle is twice the angle at the circumference.

3 **a** $a = 25°$, angles in the same segment are equal.
$b = 45°$, angles in the same segment are equal.

b $c = 44°$, angles in the same segment are equal.
$d = 46°$, the angle in a semicircle is 90° and the angles in a triangle total 180°.

c $e = 48°$, the angle at the centre of a circle is twice the angle at the circumference.
$f = 48°$, angles in the same segment are equal.

d $g = 100°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
$h = 100°$, angles in the same segment are equal.

4 **a** $a = 75°$, opposite angles in a cyclic quadrilateral total 180°.
$b = 105°$, angles on a straight line total 180°.
$c = 94°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 92°$, opposite angles in a cyclic quadrilateral total 180°.
$e = 88°$, angles on a straight line total 180°.
$f = 92°$, angles in the same segment are equal.

c $h = 80°$, alternate segment theorem.

d $g = 35°$, alternate segment theorem and the angle in a semicircle is 90°.

5 Angle BAT = x.

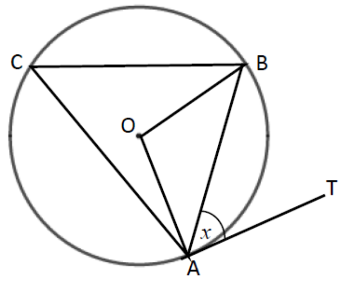
Angle OAB = $90° - x$ because the angle between the tangent and the radius is 90°.

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = $180° - (90° - x) - (90° - x) = 2x$ because angles in a triangle total 180°.

Angle ACB = $2x \div 2 = x$ because the angle at the centre is twice the angle at the circumference.



Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Corbettmaths.com videos of use:

332 - Trigonometry: 3D

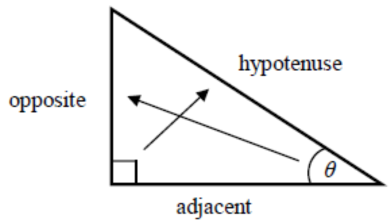
338 - Trigonometry: Sine graph

339 - Trigonometry: Cosine graph

340 - Trigonometry: Tangent graph

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

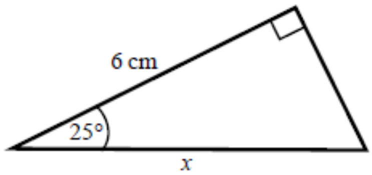
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

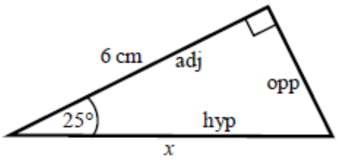
- The sine, cosine and tangent of some angles may be written exactly.

	0	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

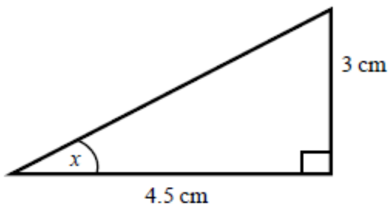
Examples

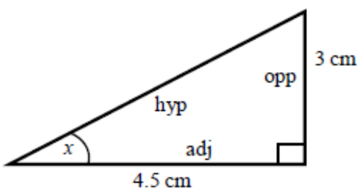
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



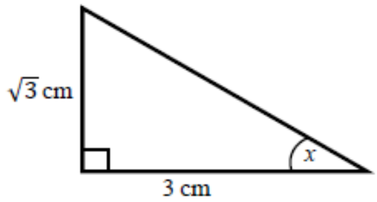
	**1** Always start by labelling the sides.
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \dfrac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \dfrac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\,267\,5\ldots$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

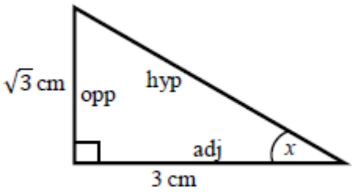
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \dfrac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\dfrac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\,067\,5\ldots$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle x.

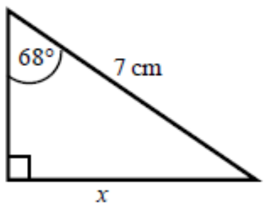


	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{\sqrt{3}}{3}$	**3** Substitute the sides and angle into the tangent ratio.
$x = 30°$	**4** Use the table from the key points to find the angle.

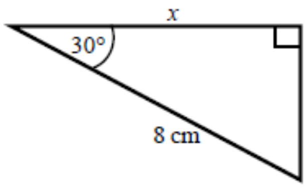
Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

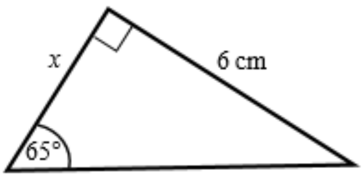
a



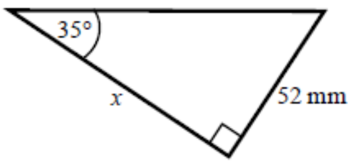
b



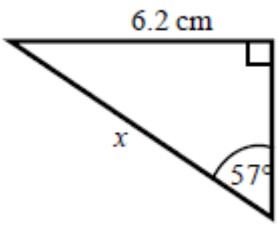
c



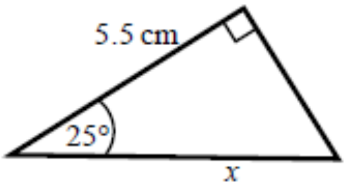
d



e



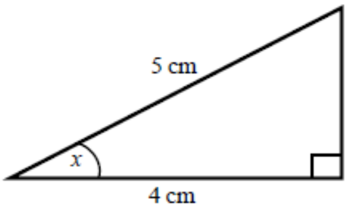
f



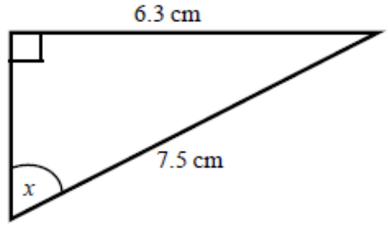
edexcel

2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

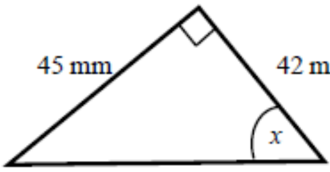
a



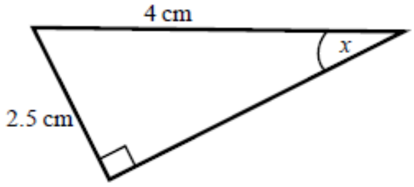
b



c

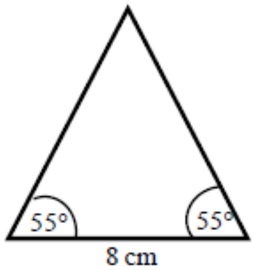


d



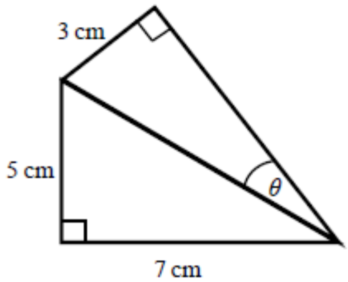
3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



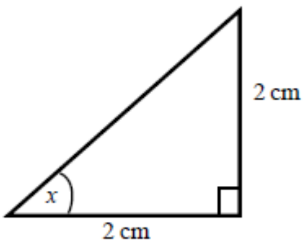
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.

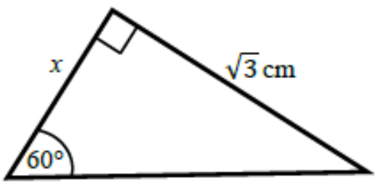


5 Find the exact value of x in each triangle.

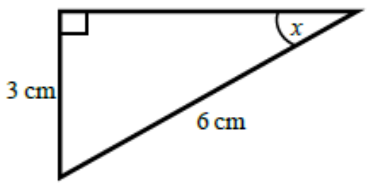
a



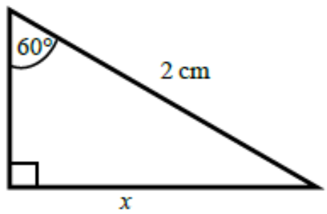
b



c



d



Pearson

edexcel

The cosine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

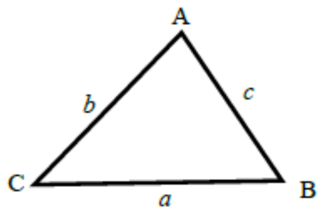
Corbettmaths.com videos of use:

335 - Trigonometry: cosine rule (sides)

336 - Trigonometry: cosine rule (angles)

Key points

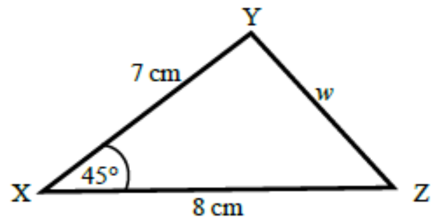
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

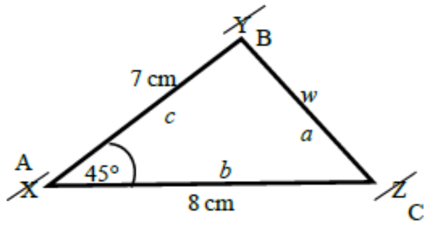


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Examples

Example 4 Work out the length of side w.
Give your answer correct to 3 significant figures.





$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$

$w^2 = 33.804\,040\,51...$
$w = \sqrt{33.80404051}$
$w = 5.81$ cm

1 Always start by labelling the angles and sides.

2 Write the cosine rule to find the side.

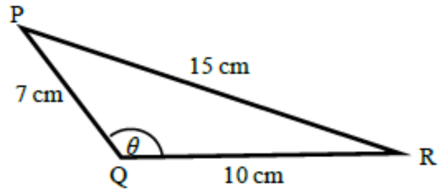
3 Substitute the values a, b and A into the formula.

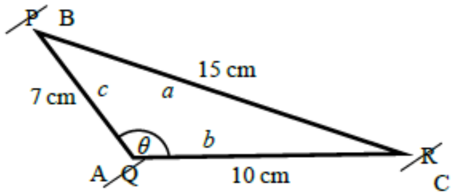
4 Use a calculator to find w^2 and then w.

5 Round your final answer to 3 significant figures and write the units in your answer.

Pearson

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

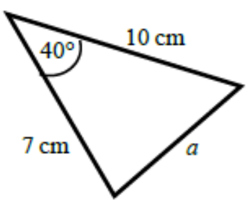


	**1** Always start by labelling the angles and sides.
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values a, b and c into the formula.
$\cos\theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\ 349...$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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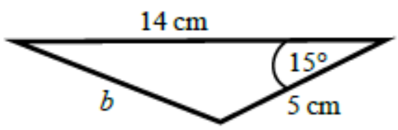

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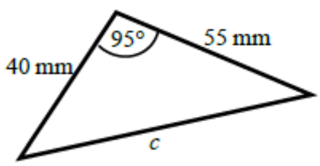

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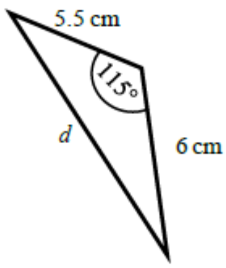
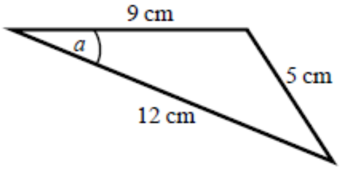


7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



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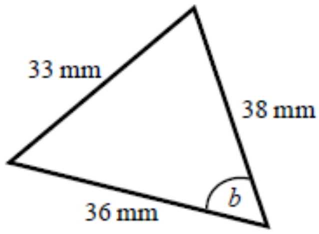
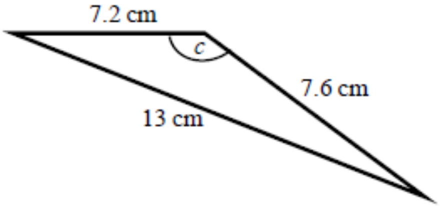
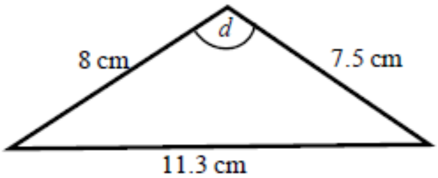


c

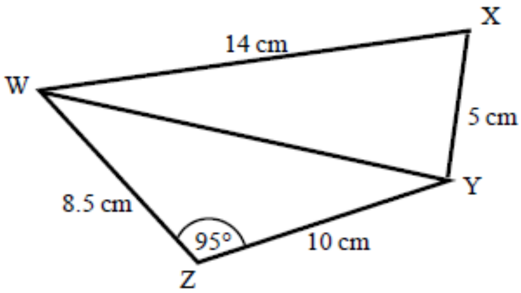


d



8 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.



The sine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

Corbettmaths.com videos of use:

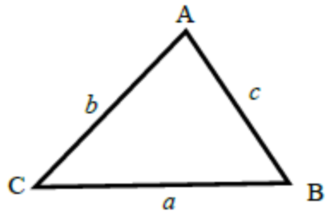
333 - Trigonometry: sine rule (sides)

334 - Trigonometry: sine rule (angles)

334a - Trigonometry: sine rule (ambiguous case)

Key points

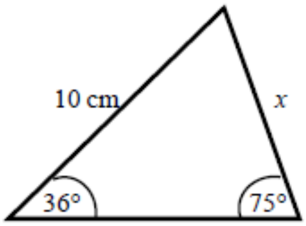
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Examples

Example 6 Work out the length of side x.
Give your answer correct to 3 significant figures.



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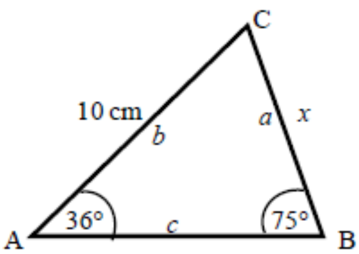

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{x}{\sin 36°} = \dfrac{10}{\sin 75°}$

$x = \dfrac{10 \times \sin 36°}{\sin 75°}$

$x = 6.09$ cm

</td>
<td>

1 Always start by labelling the angles and sides.

2 Write the sine rule to find the side.

3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make x the subject.

5 Round your answer to 3 significant figures and write the units in your answer.

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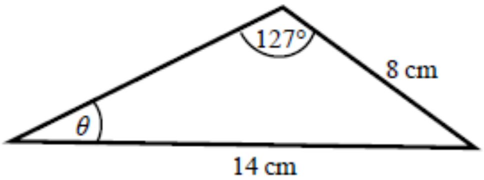
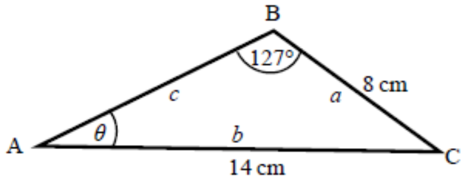
Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

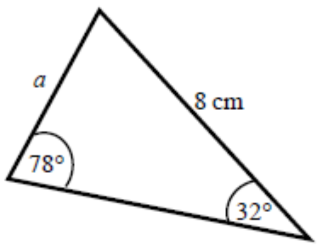


	**1** Always start by labelling the angles and sides.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$	**3** Substitute the values a, b, A and B into the formula.
$\sin \theta = \frac{8 \times \sin 127°}{14}$	**4** Rearrange to make $\sin \theta$ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

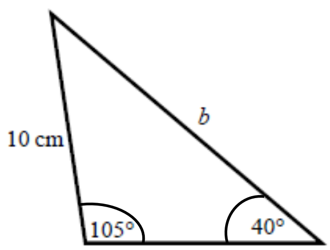
Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

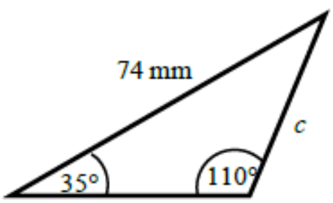
a



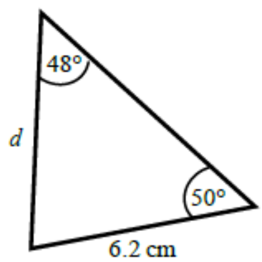
b



c

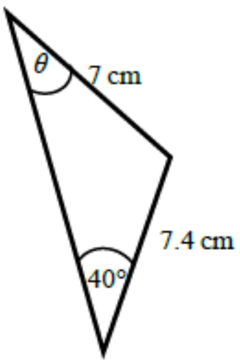


d



10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



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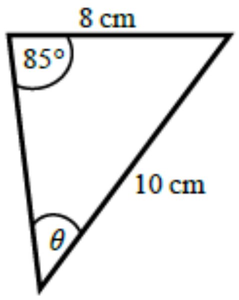

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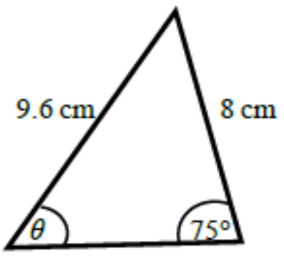
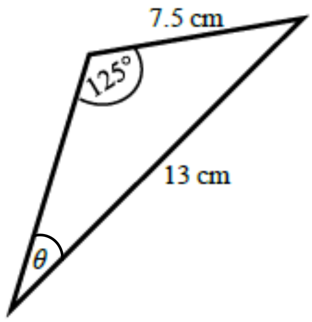
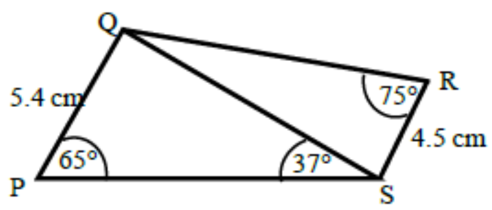


d



11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



Areas of triangles

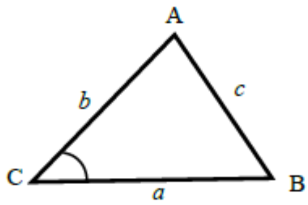
A LEVEL LINKS **Scheme of work:** 4a. Trigonometric ratios and graphs **Textbook:** Pure Year 1, 9.3 Areas of triangles

Corbettmaths.com videos of use:

337 – Area of a triangle

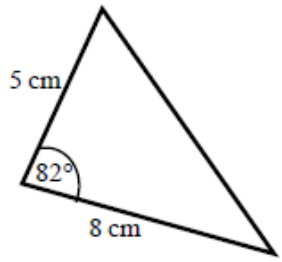
Key points

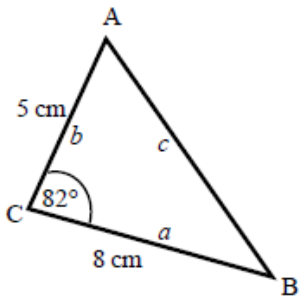
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.

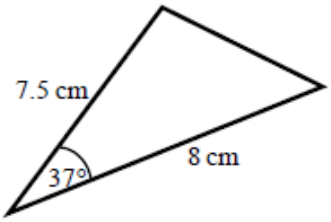


	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$	**3** Substitute the values of a, b and C into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

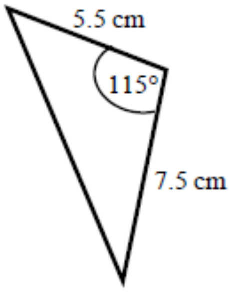
Practice

12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

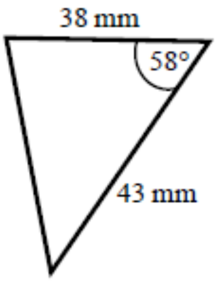
a



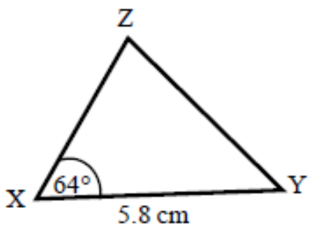
b



c



13 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



Hint:

Rearrange the formula to make a side the subject.

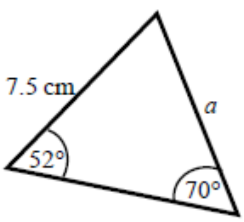
Extend

14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

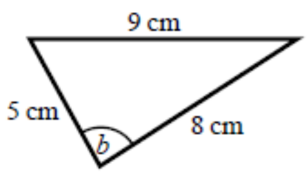
Hint:

For each one, decide whether to use the cosine or sine rule.

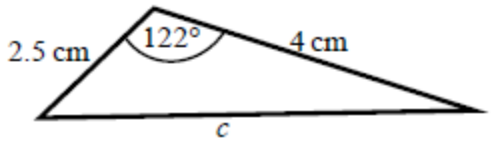
a



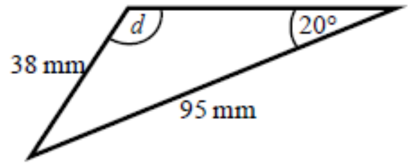
b



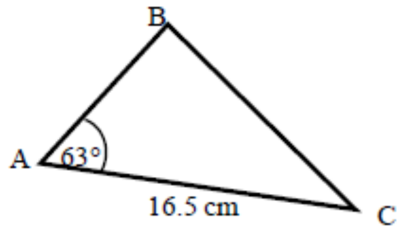
c



d



15 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°
3	5.71 cm							
4	20.4°							
5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm
6	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
7	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
8	**a**	13.7 cm	**b**	76.0°				
9	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
10	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
11	**a**	8.13 cm	**b**	32.3°				
12	**a**	18.1 cm^2	**b**	18.7 cm^2	**c**	693 mm^2		
13	5.10 cm							
14	**a**	6.29 cm	**b**	84.3°	**c**	5.73 cm	**d**	58.8°
15	15.3 cm							

Rearranging equations

A LEVEL LINKS **Scheme of work:** 6a. Definition, differentiating polynomials, second derivatives **Textbook:** Pure Year 1, 12.1 Gradients of curves

Corbettmaths.com videos of use:

111 - Equations: involving fractions

112 - Equations: fractional advanced

112 - Equations: cross multiplication

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \dfrac{v-u}{a}$	**1** Get the terms containing t on one side and everything else on the other side. **2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \dfrac{r}{2-\pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\dfrac{t+r}{5} = \dfrac{3t}{2}$.

$\dfrac{t+r}{5} = \dfrac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \dfrac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

$r = \dfrac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t + 5$ $rt - 3t = 5 + r$ $t(r-3) = 5 + r$ $t = \dfrac{5+r}{r-3}$	**1** Remove the fraction first by multiplying throughout by $t - 1$. **2** Expand the brackets. **3** Get the terms containing t on one side and everything else on the other side. **4** Factorise the LHS as t is a common factor. **5** Divide throughout by $r - 3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \dfrac{S}{T}$ [T]

4 $p = \dfrac{q-r}{t}$ [t]

5 $u = at - \dfrac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ [y]

8 $x = \dfrac{2a-1}{3-a}$ [a]

9 $x = \dfrac{b-c}{d}$ [d]

10 $h = \dfrac{7g-9}{2+g}$ [g]

11 $e(9 + x) = 2e + 1$ [e]

12 $y = \dfrac{2x+3}{4-x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \dfrac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \dfrac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\dfrac{xy}{z} = \dfrac{ab}{cd}$ **b** $\dfrac{4\pi cx}{d} = \dfrac{3z}{py^2}$

15 Make sin B the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

16 Make cos B the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

a $\dfrac{p}{q}(sx + t) = x - 1$ **b** $\dfrac{p}{q}(ax + 2y) = \dfrac{3p}{q^2}(x - y)$

Answers

1 $d = \dfrac{C}{\pi}$

2 $w = \dfrac{P - 2l}{2}$

3 $T = \dfrac{S}{D}$

4 $t = \dfrac{q - r}{p}$

5 $t = \dfrac{2u}{2a - 1}$

6 $x = \dfrac{V}{a + 4}$

7 $y = 2 + 3x$

8 $a = \dfrac{3x + 1}{x + 2}$

9 $d = \dfrac{b - c}{x}$

10 $g = \dfrac{2h + 9}{7 - h}$

11 $e = \dfrac{1}{x + 7}$

12 $x = \dfrac{4y - 3}{2 + y}$

13 **a** $r = \sqrt{\dfrac{A}{\pi}}$ **b** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

c $r = \dfrac{P}{\pi + 2}$ **d** $r = \sqrt{\dfrac{3V}{2\pi h}}$

14 **a** $x = \dfrac{abz}{cdy}$ **b** $x = \dfrac{3dz}{4\pi cpy^2}$

15 $\sin B = \dfrac{b \sin A}{a}$

16 $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

17 **a** $x = \dfrac{q + pt}{q - ps}$ **b** $x = \dfrac{3py + 2pqy}{3p - apq} = \dfrac{y(3 + 2q)}{3 - aq}$

Volume and surface area of 3D shapes

A LEVEL LINKS
Scheme of work: 6b. Gradients, tangents, normals, maxima and minima

Corbettmaths.com videos of use:

359 - Volume: cone

360 - Volume: pyramid

360a - Volume: Frustum

361 - Volume: sphere

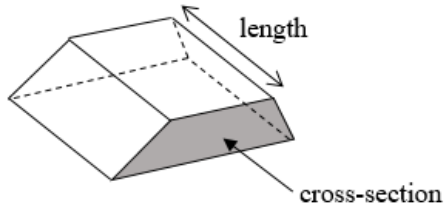
313 - Surface area: sphere

314 - Surface area: cone

315 - Surface area: cylinders

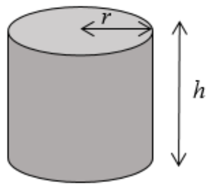
Key points

- Volume of a prism = cross-sectional area × length.
- The surface area of a 3D shape is the total area of all its faces.

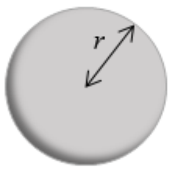


- Volume of a pyramid = $\frac{1}{3}$ × area of base × vertical height.

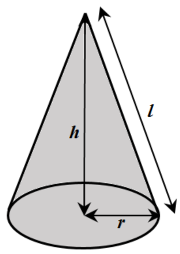
- Volume of a cylinder = $\pi r^2 h$
- Total surface area of a cylinder = $2\pi r^2 + 2\pi rh$



- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Surface area of a sphere = $4\pi r^2$

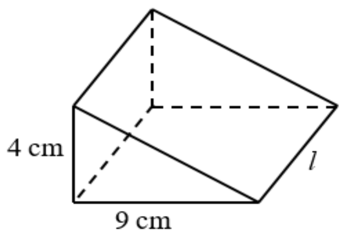


- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Total surface area of a cone = $\pi rl + \pi r^2$



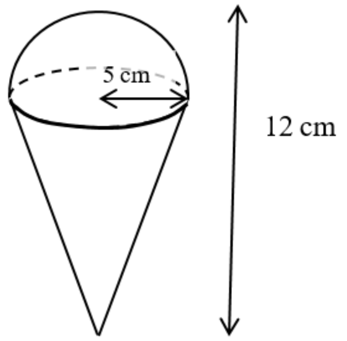
Examples

Example 1 The triangular prism has volume 504 cm^3.
Work out its length.



$V = \frac{1}{2}bhl$ $504 = \frac{1}{2} \times 9 \times 4 \times l$ $504 = 18 \times l$ $l = 504 \div 18$ $= 28$ cm	**1** Write out the formula for the volume of a triangular prism. **2** Substitute known values into the formula. **3** Simplify **4** Rearrange to work out l **5**

Example 2 Calculate the volume of the 3D solid.
Give your answer in terms of π.

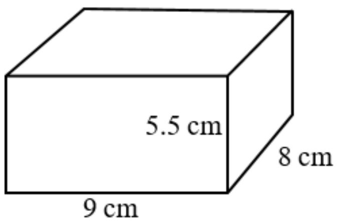


Total volume = volume of hemisphere + Volume of cone $= \frac{1}{2}$ of $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$	**1** The solid is made up of a hemisphere radius 5 cm and a cone with radius 5 cm and height 12 − 5 = 7 cm.
Total volume $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$ $+ \frac{1}{3} \times \pi \times 5^2 \times 7$	**2** Substitute the measurements into the formula for the total volume.
$= \frac{425}{3}\pi$ cm^3	**3** Remember the units.

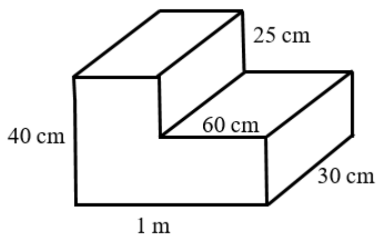
Practice

1 Work out the volume of each solid.
Leave your answers in terms of π where appropriate.

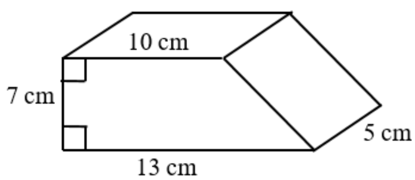
a



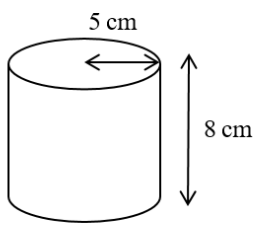
b



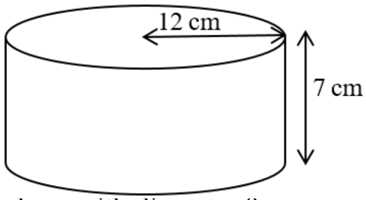
c



d



e

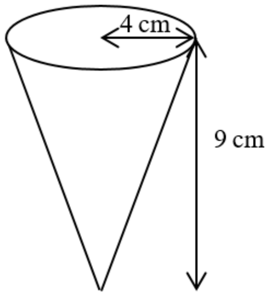


f a sphere with radius 7 cm

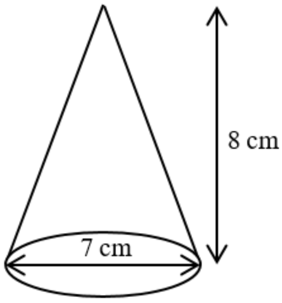
g a sphere with diameter 9 cm

h a hemisphere with radius 3 cm

i

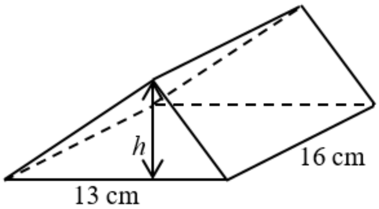


j



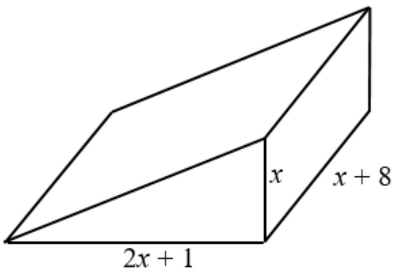
2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm^3.
Work out its length.

3 The triangular prism has volume 1768 cm^3.
Work out its height.

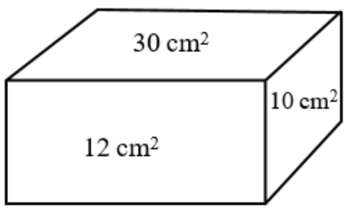


Extend

4 The diagram shows a solid triangular prism.
All the measurements are in centimetres.
The volume of the prism is V cm^3.
Find a formula for V in terms of x.
Give your answer in simplified form.



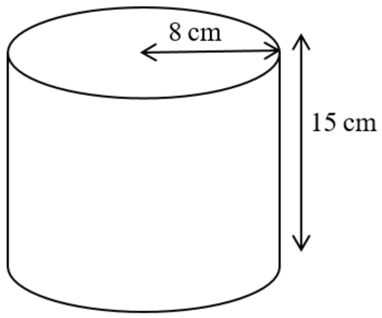
5 The diagram shows the area of each of three faces of a cuboid.



The length of each edge of the cuboid is a whole number of centimetres.

Work out the volume of the cuboid.

6 The diagram shows a large catering size tin of beans in the shape of a cylinder.



The tin has a radius of 8 cm and a height of 15 cm.

A company wants to make a new size of tin.

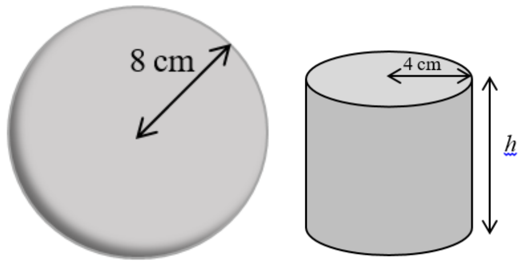
The new tin will have a radius of 6.7 cm.

It will have the same volume as the large tin.

Calculate the height of the new tin.

Give your answer correct to one decimal place.

7 The diagram shows a sphere and a solid cylinder.



The sphere has radius 8 cm.

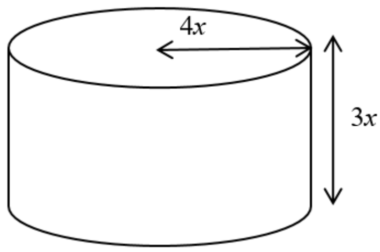
The solid cylinder has a base radius of 4 cm and a height of h cm.

The total surface area of the cylinder is half the total surface area of the sphere.

Work out the ratio of the volume of the sphere to the volume of the cylinder.

Give your answer in its simplest form.

8 The diagram shows a solid metal cylinder.



The cylinder has base radius $4x$ and height $3x$.

The cylinder is melted down and made into a sphere of radius r.

Find an expression for r in terms of x.

Answers

1 **a** $V = 396\ \text{cm}^3$ **b** $V = 75\ 000\ \text{cm}^3$

c $V = 402.5\ \text{cm}^3$ **d** $V = 200\pi\ \text{cm}^3$

e $V = 1008\pi\ \text{cm}^3$ **f** $V = \frac{1372}{3}\pi\ \text{cm}^3$

g $V = 121.5\pi\ \text{cm}^3$ **h** $V = 18\pi\ \text{cm}^3$

i $V = 48\pi\ \text{cm}^3$ **j** $V = \frac{98}{3}\pi\ \text{cm}^3$

2 17 cm

3 17 cm

4 $V = x^3 + \frac{17}{2}x^2 + 4x$

5 $60\ \text{cm}^3$

6 21.4 cm

7 32 : 9

8 $r = \sqrt[3]{36x}$

Area under a graph

Corbettmaths.com videos of use:

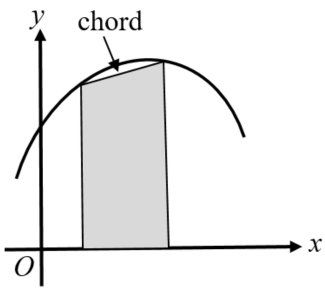
389 - Area under a Graph

390a - Instantaneous Rate of Change

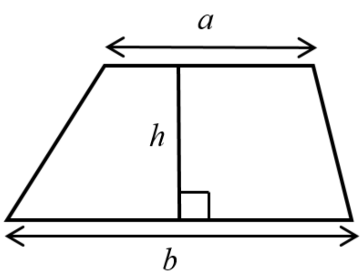
A LEVEL LINKS
Scheme of work: 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

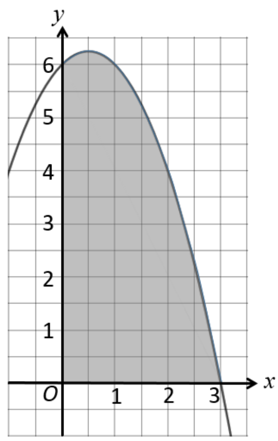


- The area of a trapezium = $\frac{1}{2}h(a+b)$



Examples

Example 1 Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.



x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6,\ b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6,\ b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4,\ a_3 = 0 - 0 = 0$

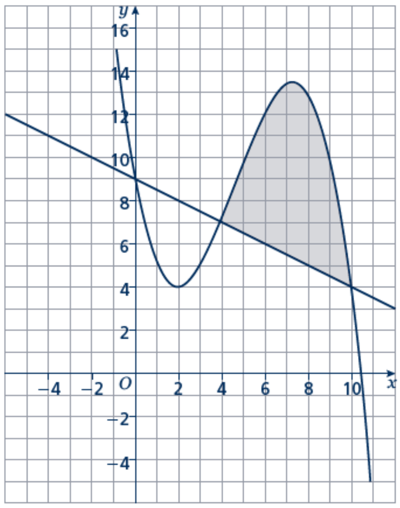
1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

(continued on next page)

$\frac{1}{2}h(a_1+b_1)=\frac{1}{2}\times 1(6+6)=6$ $\frac{1}{2}h(a_2+b_2)=\frac{1}{2}\times 1(6+4)=5$ $\frac{1}{2}h(a_3+b_3)=\frac{1}{2}\times 1(4+0)=2$ Area = 6 + 5 + 2 = 13 units^2	**3** Work out the area of each trapezium. h = 1 since the width of each trapezium is 1 unit. **4** Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.



<table><tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>7</td><td>12</td><td>13</td><td>4</td></tr></table> <table><tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>7</td><td>6</td><td>5</td><td>4</td></tr></table>	**1** Use a table to record y on the curve for each value of x. **2** Use a table to record y on the straight line for each value of x.
Trapezium 1: $a_1=7-7=0,\ b_1=12-6=6$ Trapezium 2: $a_2=12-6=6,\ b_2=13-5=8$ Trapezium 3: $a_3=13-5=8,\ a_3=4-4=0$	**3** Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.
$\frac{1}{2}h(a_1+b_1)=\frac{1}{2}\times 2(0+6)=6$ $\frac{1}{2}h(a_2+b_2)=\frac{1}{2}\times 2(6+8)=14$ $\frac{1}{2}h(a_3+b_3)=\frac{1}{2}\times 2(8+0)=8$ Area = 6 + 14 + 8 = 28 units^2	**4** Work out the area of each trapezium. h = 2 since the width of each trapezium is 2 units. **5** Work out the total area. Remember to give units with your answer.

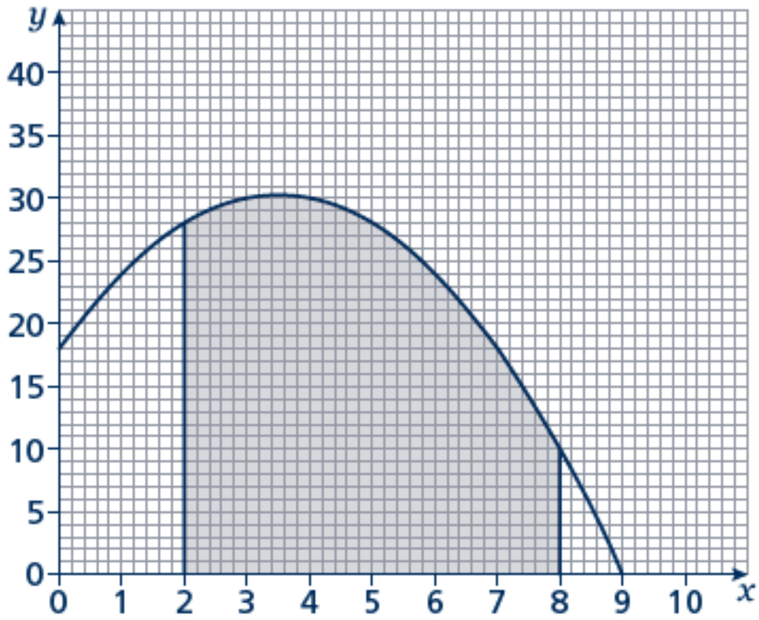
Practice

Hint:

For a full answer, remember to include 'units^2'.

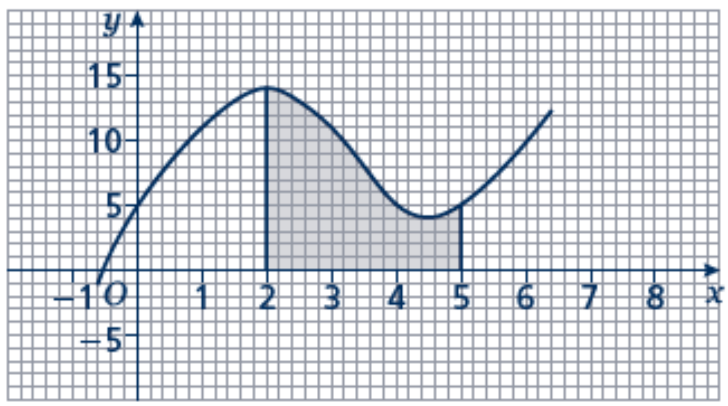
1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



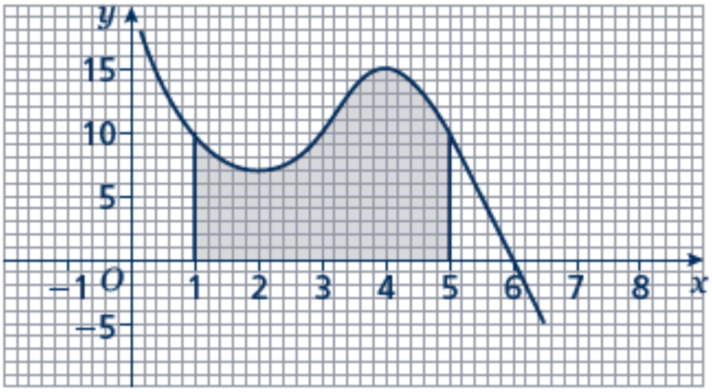
3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.



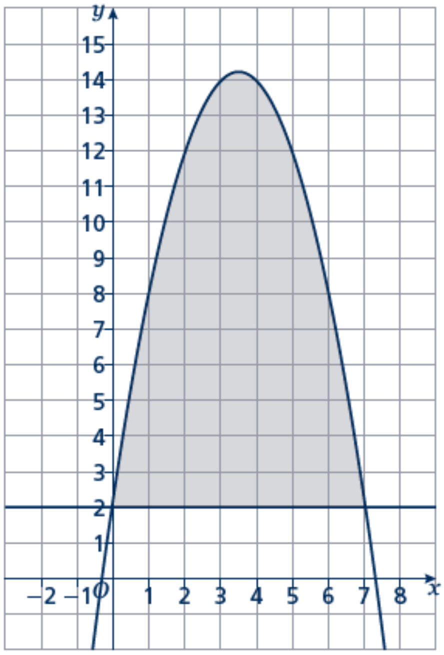
5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

6 Estimate the shaded area.
Use four strips of equal width.



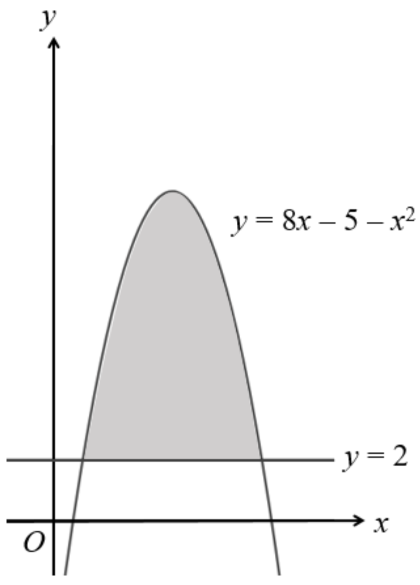
7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
Use seven strips of equal width.

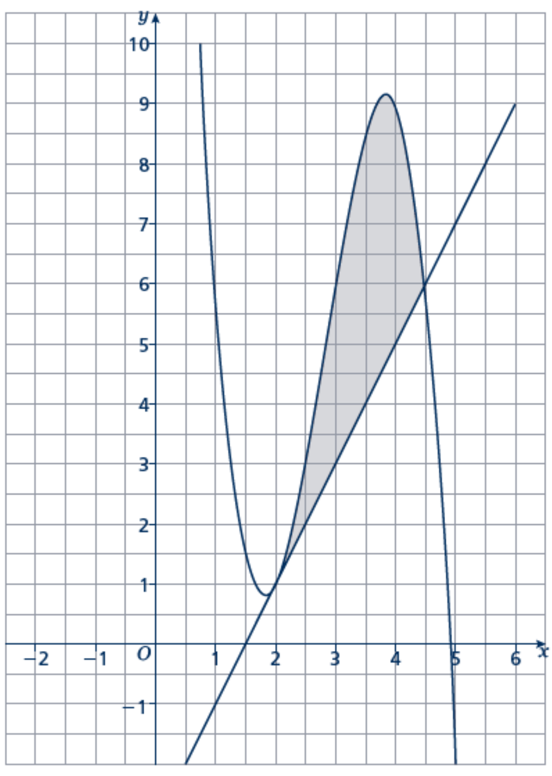


Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.



Answers

1 34 units^2

2 149 units^2

3 14 units^2

4 $25\frac{1}{4}$ units^2

5 35 units^2

6 42 units^2

7 $26\frac{7}{8}$ units^2

8 56 units^2

9 35 units^2

10 $6\frac{1}{4}$ units^2